

to the first Vlasov-type equation:

$$[E\delta^3/12(1-\mu^2)]\Delta\Delta w + (1/1-\mu)\Delta M_T - D\Phi = q \quad (7)$$

where

$$D = \frac{1}{A_1 A_2} \left[\frac{\partial}{\partial \alpha_1} \left(\frac{1}{R_2} \frac{A_2}{A_1} \frac{\partial}{\partial \alpha_1} \right) + \frac{\partial}{\partial \alpha_2} \left(\frac{1}{R_1} \frac{A_1}{A_2} \frac{\partial}{\partial \alpha_2} \right) \right]$$

The function Φ is not arbitrary, since it must lead to displacements that are compatible. The condition of compatibility is given by (Ref. 3, p. 28)

$$\begin{aligned} \frac{k_1}{R_2} + \frac{k_2}{R_1} + \frac{1}{A_1 A_2} \left\{ \frac{\partial}{\partial \alpha_1} \frac{1}{A_1} \left[A_2 \frac{\partial \epsilon_2}{\partial \alpha_1} + \frac{\partial A_2}{\partial \alpha_1} (\epsilon_2 - \epsilon_1) - \frac{A_1}{2} \frac{\partial w}{\partial \alpha_2} - \frac{\partial A_1}{\partial \alpha_2} \omega \right] + \frac{\partial}{\partial \alpha_2} \frac{1}{A_2} \left[A_1 \frac{\partial \epsilon_1}{\partial \alpha_2} + \frac{\partial A_1}{\partial \alpha_2} (\epsilon_1 - \epsilon_2) - \frac{A_2}{2} \frac{\partial \omega}{\partial \alpha_1} - \frac{\partial A_2}{\partial \alpha_1} \omega \right] \right\} = 0 \quad (8) \end{aligned}$$

Substitution of Eqs. (2, 4, and 6) into Eq. (8) then yields

$$E\delta D w + \Delta\Delta\Phi + \Delta N_T = 0 \quad (9)$$

where some higher order terms have been neglected. Eqs. (7) and (9), together with appropriate boundary conditions,⁴ can be used to obtain approximate solutions for thermoelastic problems of thin shells.

References

- 1 Bijlaard, P. P., "Differential equations for cylindrical shells with arbitrary temperature distribution," *J. Aerospace Sci.* **25**, 594-595 (1958).
- 2 Boley, B. A. and Weiner, J. H., *Theory of Thermal Stresses* (John Wiley & Sons Inc., New York, 1960), pp. 379-384.
- 3 Novozhilov, V. V., *The Theory of Thin Shells* (P. Noordhoff Ltd., Groningen, Netherlands, 1959), Chap. I.
- 4 Goldenveizer, A. L., *Theory of Elastic Thin Shells* (Pergamon Press, New York, 1961), p. 106.

Total Heating Load on Blunt Axisymmetric Bodies in Low-Density Flow

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Nomenclature

\dot{q}_{0f_m}	= heat transfer rate at stagnation point
	= $\rho_\infty U_\infty^3/2$ = approximate stagnation point rate for free molecular flow
\dot{q}_{avg}	= average heat transfer rate based on surface area
\dot{q}_{avg, f_m}	= average rate for free molecular flow
U_∞	= velocity of freestream
ρ_∞	= density of freestream
R	= radius of body \equiv radius of curvature for hemisphere
Re_2	= $\rho_\infty U_\infty R / \mu_2$
μ_2	= viscosity immediately downstream of normal shock
H_0	= total enthalpy of freestream
H_w	= enthalpy corresponding to body surface conditions
$(dU/ds)_0$	= "inviscid" velocity gradient at stagnation point

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THIS is a report of total heating rates of blunt, axisymmetric noses in a low-density, hypersonic wind tunnel. Minimum stream density was such that Knudsen number based on nose radius and conditions immediately behind the normal portion of the bow shock exceeded 0.1. Thus, scaling on the basis of Knudsen number, a body of 1-ft nose radius at a maximum altitude above 315,000 ft was simulated.

The LDH wind tunnel,¹ in operation at the von Kármán Gas Dynamics Facility of the Arnold Engineering Development Center, was used to obtain the data presented. Although calorimetry data show thermodynamic equilibrium to exist at the throat of the nozzle, computations² show molecular vibration to be essentially frozen downstream of the throat when nitrogen is the medium. A similar calculation indicates that vibration remains frozen throughout the shock layer, but temperatures corresponding to the active and inert degrees of freedom are nearly equal on the stagnation streamline immediately downstream of the bow shock. It is assumed that the portion of the total enthalpy represented by vibration in the present case contributes to heating of the test bodies. Inasmuch as the total temperature was below that at which nitrogen dissociates, no recombination chemistry was involved. When argon was used as the medium, it was assumed that it behaved as a perfect gas, although excitation to a metastable state occurred.

Talbot³ has shown that the electrical potential of a probe in an ionized stream is an important factor in determining the heat transfer to the probe. As a precaution, all the results presented herein were obtained with the probe grounded with respect to the tunnel wall.

The models consisted of hemisphere-cylinders and flat-faced cylinders. Total heat flux (Btu/sec) to the noses was measured. This was converted to average heat transfer rate per unit area, \dot{q}_{avg} Btu/ft²-sec, by dividing total heat flux by the wetted area of the nose. Descriptions of the models and other details may be found in a test report.⁴

A problem arose in the comparison of the measured average values with theories presented for stagnation point heat transfer because the theoretical distribution appropriate to the flow conditions is not available in all cases. This left no recourse except the assumption that one of the theories for thin boundary layers (high Reynolds numbers) may be used to obtain the relation between average and stagnation point heating rates at very low Reynolds numbers. This was done by assuming that Lees' distribution⁵ was valid for the case of the hemispheres. For the case of the flat-nosed models, the distribution computed by Vinokur⁶ was used. The relations inferred from these distributions are

Nose shape	\dot{q}_0	$\dot{q}_{0f_m} = \rho_\infty U_\infty^3/2$
Hemisphere ⁵	2.50 \dot{q}_{avg}	2.00 \dot{q}_{avg, f_m}
Flat face ⁶	0.756 \dot{q}_{avg}	\dot{q}_{avg, f_m}

The experimental results are presented in Fig. 1 and, in the case of the hemisphere, compared with theories for low-density flow.

Behavior of the data appears qualitatively in agreement with results of the most appropriate theories. There is an indication that the data at the lowest values of Re_2 on Fig. 1a depart from the extrapolated, theoretically derived curves. First, it should be noted that the data extend to Reynolds numbers lower than are compatible with the flow models assumed for theoretical analysis. Second, the earlier remarks on the relation of average rates to stagnation point rates may be relevant. The hemisphere tested in argon yielded results in good agreement with theory, as shown in Fig. 1b.

The constant-density, subsonic flow field on which the heating rate distribution of Ref. 6 is based cannot be valid at Reynolds numbers where a fully merged shock layer exists for a highly cooled body. Thus, some of the difference between theory and experiment seen in Fig. 1c would be expected for this reason.

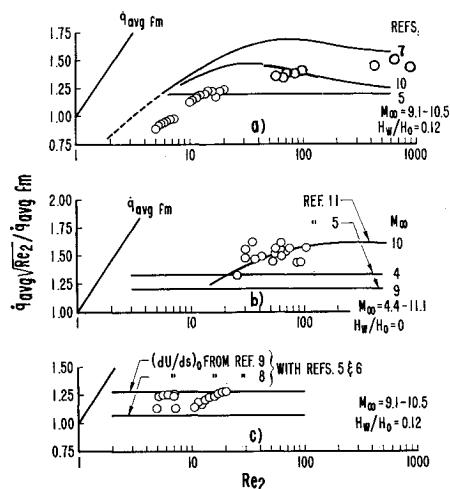


Fig. 1 Average heat-transfer rate parameter: a) hemisphere in nitrogen; b) hemisphere in argon; c) flat-face in nitrogen

References

- ¹ Potter, J. L., Kinslow, M., Arney, G. D., Jr., and Bailey A. B., "Initial results from a low density hypervelocity wind tunnel," *Hypersonic Flow Research*, edited by F. R. Riddell (Academic Press, New York, 1962), pp. 599-624.
- ² Stollery, J. L., Smith, J. E., and Park, C., "The effects of vibrational relaxation on hypersonic nozzle flows," AGARD Hypersonics Conference, Brussels (1962).
- ³ Talbot, L., "Theory of the stagnation-point Langmuir probe," *Phys. Fluids* **3**, 289-298 (1960).
- ⁴ Potter, J. L. and Miller, J. T., "Experimental heat transfer to blunt axisymmetric bodies near the limit of continuum flow," *Arnold Eng. Dev. Center TDR-62-155* (August 1962).
- ⁵ Lees, L., "Laminar heat transfer over blunt-nosed bodies at hypersonic flight speeds," *Jet Propulsion* **26**, 259-269 (1956).
- ⁶ Vinokur, M., "Laminar heat-transfer distribution on oblate ellipsoidal noses in hypersonic flow," *J. Aerospace Sci.* **29**, 113-114 (1962).
- ⁷ Cheng, H. K., "Hypersonic shock-layer theory of the stagnation region at low Reynolds number," Cornell Aeronaut. Lab. Rept. AF-1285-A-7 (April 1961).
- ⁸ Chones, A. J., "Heat transfer and pressure measurements on flat-faced flared-tail circular cylinders and normal disks," Naval Ordnance Laboratory NAVORD Rept. 6669 (June 1959).
- ⁹ Probstein, R. F., "Inviscid flow in the stagnation point region of very blunt-nosed bodies at hypersonic flight speeds," Brown Univ. Wright Air Dev. Center TN 56-395 (September 1956).
- ¹⁰ Probstein, R. F. and Kemp, N. H., "Viscous aerodynamic characteristics in hypersonic rarefield gas flow," *J. Aerospace Sci.* **27**, 174-192 (1960).
- ¹¹ Levinsky, E. S. and Yoshihara, H., "Rarefield hypersonic flow over a sphere," *Hypersonic Flow Research*, edited by F. R. Riddell (Academic Press, New York, 1962), pp. 81-106.

Approximate Solution of the Energy Equation

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Introduction

IN a previous paper by Bush,¹ it has been shown that the incompressible and compressible laminar boundary layer equations may be solved by an approximate method, namely,

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Table 1 Wall temperature gradient

	$Pr = 0.7$	$Pr = 1.0$	$Pr = 2.0$	$Pr = 5.0$	$Pr = 10$	$Pr = 20$	
$\beta = 0$	0.408	0.467	0.589	...	1.015	1.241	approx.
$\gamma = 0$	0.407	0.470 ^a	0.582	...	0.986	1.230	exact
$\beta = 1$	0.468	0.536	...	0.949	approx.
$\gamma = 0$	0.488	0.560	...	1.011	exact
$\beta = 1.6$	0.483	0.556	approx.
$\gamma = 0$	0.506	0.583	exact
$\beta = 0$...	1.547	approx.
$\gamma = 1$...	1.625	exact
$\beta = 1$...	0.801	approx.
$\gamma = 1$...	0.812	exact

^a See Ref. 1.

an iteration procedure. It is the object of this note to extend this method to the solution of the laminar boundary layer energy equation for wedge-type flow with variable wall temperature.

Analysis

The appropriate energy equation for the type of flow and wall temperature mentioned previously is

$$\frac{d^2\theta}{d\eta^2} + Pr f \frac{d\theta}{d\eta} - Pr(2 - \beta) \gamma \frac{df}{d\eta} (\theta - 1) = 0 \quad (1)$$

with the boundary conditions

$$\eta = 0: \theta = 0 \quad \eta \rightarrow \infty: \theta = 1 \quad (2)$$

where β determines the wedge angle, γ is the power to which the wall temperature is raised, and Pr is the Prandtl number.

Denoting $df/d\eta$ by w , Eq. (1) becomes

$$\frac{d^2\theta}{d\eta^2} = -Pr \frac{d\theta}{d\eta} \int_0^\eta w d\eta_1 + Pr(2 - \beta) \gamma w(\theta - 1) \quad (3)$$

This equation may be solved by iteration by introducing a suitable approximation for θ in the right-hand side. Denoting the resulting solution by G , one has

$$\frac{d^2G}{d\eta^2} = -Pr \frac{d\theta}{d\eta} \int_0^\eta w d\eta_1 + Pr(2 - \beta) \gamma w(\theta - 1) \quad (4)$$

The first integral of this equation is

$$\frac{dG}{d\eta} = \int_\eta^\infty Pr \frac{d\theta}{d\eta_1} \left(\int_0^{\eta_1} w d\eta_2 \right) d\eta_1 - \int_\eta^\infty Pr \times (2 - \beta) \gamma w(\theta - 1) d\eta_1 \quad (5)$$

The second integral of this equation is

$$G(\eta) = \int_0^\eta d\eta_1 \left[\int_\eta^\infty Pr \frac{d\theta}{d\eta_2} \left(\int_0^{\eta_2} w d\eta_3 \right) d\eta_2 \right] - \int_0^\eta d\eta_1 \left[\int_{\eta_1}^\infty Pr (2 - \beta) \gamma w(\theta - 1) d\eta_2 \right] \quad (6)$$

satisfying the boundary condition $G(0) = 0$.

If the approximating functions w and θ are

$$w = \text{erf}(a\eta) \quad a = \text{const} \quad (7a)$$

$$\theta = \text{erf}(b\eta) \quad b = \text{const} \quad (7b)$$

then the first integration, Eq. (5), yields the temperature gradient at the wall for $\eta = 0$:

$$\frac{dG(0)}{d\eta} = Pr[1 + (2 - \beta)\gamma] \left[\frac{(1 + m^2)^{1/2} - 1}{a(\pi)^{1/2}} \right] m = \frac{a}{b} \quad (8)$$